Indian J. Phys. **44**, 425–430, (1970)

The binding energy calculation of triton with Faddeev equation in separable approximation

By Gita Purkayastha, S. N. Banerjee* and N. C. Sil

Department of Theoretical Physics,
Indian Association for the Cultivation of Science, Jadavpur, Calcutta-32

(*Received* 17 *September* 1970)

The binding energy of triton has been calculated using Faddeev equation as formulated by Lovelace. Three sets of values have been taken for the parameters occurring in the separable two-body non-local potential with one term only.

Introduction

A number of theoretical attempts have been made towards the calculation of the triton binding energy Variational and other equivalent methods (Rarita & Present 1937, Ohmura & Ohmura 1962 Wageningen & Kok 1967, Fiedelday *et al* 1968) have been employed with a variety of simple interactions. Amado (1963) and his collaborators (1965) have used a three-body model in which the composite systems have been represented by elementary particles. Treating the Schrodinger equation for the three body problem as a direct eigen value equation, Mitra (1962) has computed the triton binding energy assuming separable potential to act between pairs.

Faddeev (1961) has given a satisfactory theory of non-relativistic three particle systems. For local potentials this involves the solution of a set of coupled integral equations in at least two continuous variables If separable non-local interactions are assumed instead of local potentials, we have single variable integral equation. We also get single variable integral equation if we employ a Sturmian set of expansion for the two-body t-matrix. Kharchenko & Sitenko (1963) have used Faddeev equation with two body non-local separable potential. Kharchenko *et al* (1966) have further carried out calculations with two-body local potentials of square well and Hulthen types utilizing a Sturmian set of expansion. Malfliet & Tjon (1969) have calulated the binding energy of triton with two-body local central Yukawa interactions and they have also considered tensor force. Approximating the two-body t-matrix by one separable term after Noyes (1965) and Kowalski (1965), Humberston *et al* (1968) have obtained the binding energy of triton. Lovelace (1964) has given a practical theory of three particle states based on Faddeev's work. As pointed out by Lovelace if a partial wave is dominated by a bound state or a resonance, then the off-shell t-matrix can be approximated by a separable term. When this is used as an input in the Faddeev equation, we get a coupled equation in one variable which is amenable to numerical solution.

* Present address : Physics Department, Jadavpur University, Calcutta-32.

The separable approximation, though somewhat unrealistic, enormously simplifies both the bound state and scattering calculation. Further it provides a simple framework in which to investigate the sensitivity of three nucleon properties to the variation of particular two nucleon parameters.

In the present work, we have applied Lovelace formalism to calculate the triton binding energy. As for the intermediate two nucleon states we have considered only the deuteron bound state and used one term separable potential for this interaction. Three sets of two-body parameters have been used in our calculation and their fit to the two-body data of deuteron binding energy, triplet scattering length and recent phase shift (^3S) values of MacGregor *et al* (1968) have been studied.

Theory

Following Lovelace (1964), the three-body transition operators are defined as

$$U^{+}_{\alpha\beta}(s) = \sum_{\gamma\neq\alpha} V_\gamma - \sum_{\gamma\neq\alpha}\sum_{\delta\neq\beta} V_\gamma G(s) V_\delta$$

$$U^{-}_{\alpha\beta}(s) \quad \sum_{\delta\neq\beta} V_\delta - \sum_{\gamma\neq\alpha}\sum_{\delta\neq\beta} V_\gamma G(s) V$$

where $$G(s) = [H - sI)]^{-1},$$

H is the total Hamiltonian for three-particle system V_α is the pair potential, where α takes the values from 0 to 3 V_1, V_2 and V_3 are the potentials between particles (2, 3), (1, 3) and (1, 2), respectively and V_0 is equal to 0.
Therefore

$$H = H_0 + V_1 + V_2 + V_3$$

where H_0 is the free Hamiltonian for the three-body system and $G_0(s) = [H_0 - sI]^{-1}$ is the resolvent of the free Hamiltonian

The Hamiltonian of the various subsystems are

$$H_\alpha = H_0 + V_\alpha$$

The Green's functions of these Hamiltonians are

$$G_\alpha(s) = [H_\alpha - sI]^{-1}$$

These satisfy the second resolvent identity

$$G(s) = G_\beta(s) - \sum_{\delta\neq\beta} G(s) V_\delta G_\beta(s)$$

$$= G_\alpha(s) - \sum_{\gamma\neq\alpha} G_\alpha(s) V_\alpha G(s)$$

We shall define scattering amplitudes for bound states and resonances by

$$X_{\alpha n,\,\beta m}(s) = \langle \alpha n \,|\, G_0(s) U^{+}_{\alpha\beta}(s) G_0(s) \,|\, \beta m \rangle$$
$$- Z_{\alpha n,\,\beta m}(s)[1 + \lambda_{\beta m} \langle \beta m \,|\, G_0(s) \,|\, \beta m \rangle] \qquad \ldots (1)$$

If we take the potentials for bound-state scattering as

$$<q_\alpha|\, Z_{\alpha n,\,\beta m}(s)\,|q_\beta> = (1-\delta_{\alpha\beta}) < \alpha n\,|\,G_0(s)\,|\beta m> \qquad \ldots\ (2)$$

then the equation for $X_{\alpha n,\,\beta m}(s)$ becomes

$$X_{\alpha n,\,\beta m}(s) = -Z_{\alpha n,\,\beta m}(s) - \sum_{\gamma r} X_{\alpha n,\,\gamma r}(s)\hat{\tau}_{\gamma r}(s) Z_{\gamma r,\,\beta m}(s) \qquad \ldots\ (3)$$

For the three-body bound state problem, the equation (2) becomes homogeneous integral equation For identical particles and triplet two-body state we now have

$$<q_1|\, X_{nm}(s)\,|q_1> = -<q_1|\,\hat{\tau}_r(s)\,|q_1'><q_1'|\,2Z_{rm}(s)\,|q_2>\cdot \cdot<q_2|\,X_{rn}(s)\,|q_1> \qquad \ldots\ (4)$$

This homogeneous integral equation should have a solution at an energy equal to binding energy of the three-nucleon system. For equal mass particle Z_{rm} reduces to

$$<q_1|\, Z_{rm}(s)\,|q_2> = (1-\delta_{rm})\,\frac{g_r(p_1)\,g_m(p_2)}{p_2^2+q_2^2-s-i\epsilon}\cdot\frac{8}{3\sqrt{3}}$$

$$= \overset{IS}{\Lambda}_{rm}\cdot\frac{1}{2}\int_{-1}^{+1}\frac{8}{3\sqrt{3}}\cdot P_l(\cos\theta)\;\frac{g_r(p_1)g_m(p_2)}{p_2^2+q_2^2-s-i\epsilon}\,d\cos\theta \qquad \ldots\ (5)$$

where $\overset{IS}{\Lambda}_{rm}$ expresses the dependence of $Z_{rm}(s)$ on spin and isospin. The subscripts r and m in equation (5) each have only one value for triplet two body state. The superscripts I, S stand for total isotopic spin and spin of the three-body system. For triton $I = S = \frac{1}{2}$

Assuming the two-body S-wave potential in momentum space to be non-local and separable of Yamaguchi (1954) form,

$$V(p, p') = \lambda g(p) g(p')$$

We may write the two-body S-wave T-matrix which satisfies the Lippmann-Schwinger equation as

$$T(p, p'; s) = g(p)\,g(p')t(s)$$

where

$$t(s) = \left[\frac{1}{\lambda} + 4\pi\int_0^\infty \frac{q^2 dq g^2(q)}{q^2-s-i\epsilon}\right]^{-1}$$

The bound state form factor $g(p)$ is taken as

$$g(p) = \frac{N_d}{(p^2+\mu_d^2)}$$

The form factor $g(p)$ is normalized so that

$$4\pi\int_0^\infty \frac{g^2(p)p^2dp}{(p^2+E_d)^2} = 1$$

The separable potential will have a bound state if there is a point $s = -E_d$ for which

$$t(s) = \frac{1}{s+E_D} \left[4\pi \int_0^\infty \frac{q^2 dq g^2(q)}{(q^2+E_d)(q^2-s-i\epsilon)} \right]^{-1}$$

The operator $\tau(S)$ is given by the relation

$$<q_1 | \tau(s) | q_1'> = \delta_3(q_1-q_1')\, t(s-q_1^2)$$

Results and Discussions

Transforming the integration variable of equation (4) suitably so as to change the limits of the integral to ± 1 and using the usual Gaussian quadrature formula for integration, we recast the integral equation (4) to the following matrix equation

$$K_{ij}(E_T) X_j(E_T) = X_j(E_T)$$

where $K_{ij}(E_T)$ is the Kernel of the integral equation for triton binding energy E_T. We have searched for the poles of the corresponding inverse operator $[I-K_{ij}(E_T)]^{-1}$ by finding out the zero of the corresponding determinant. This energy for which the determinant vanishes, is the required binding energy of triton. We have calculated triton binding energy for three different sets of values of the two body parameters N_d and μ_d and have studied the sensitivity of triton binding energy with these parameters. The best fit values of the two body parameters occurring in the t-matrix which is used as input in Faddeev equation, give the binding energy 7.88 Mev as compared with the experimental value 8.48 Mev.

In figures 1, 2 and 3 we have plotted the theoretical triplet phase shift values as well as the absolute errors against laboratory energy for the three sets of parameters.

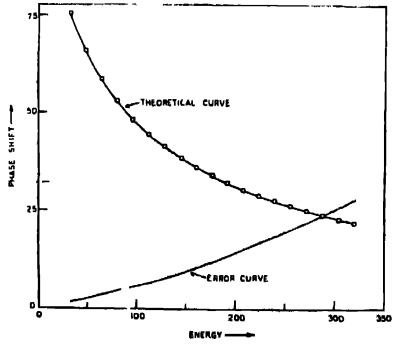

Figure 1. Set 1.

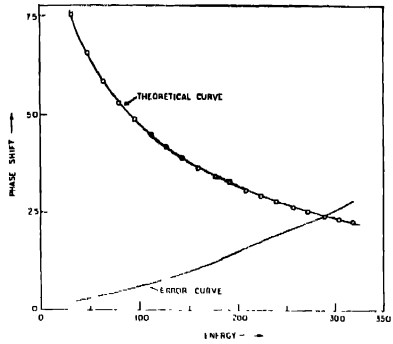

Figure 2. Set 2.

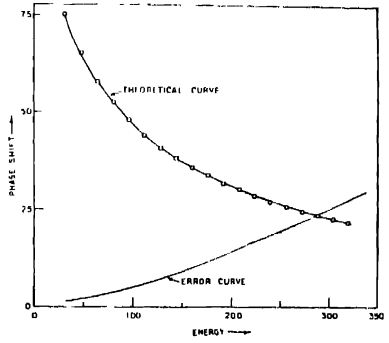

Figure 3. Set 3.

To calculate the absolute errors, we have computed ΣR^2 for the three sets where

$$\Sigma R^2 = \sum^{19} [\delta_l^{expt}(E_i) - \delta_l^{theo}(E_i)]^2,$$

$\delta_l^{expt}(E_i)$ and $\delta_l^{theo}(E_i)$ are the experimental and theoretical values of phase shifts respectively. We have taken the recent experimental phase shifts data of Mac Gregor *et al* (1968). ΣR^2 is the sum of the squares of the residuals $[\delta_l^{expt}(E_i) - \delta_l^{theo}(E_i)]$ at the 19 data points in the range 0 to 320 Mev for E_i. We have three values of ΣR^2 for the three sets of values of the two-body parameters. The minimum value of ΣR^2 obviously corresponds to the best fit of the value of the

two-body parameters. In table 1, we have enlisted the three binding energy values of triton and the absolute error values ΣR^2 for the three sets. It is seen that the best values of two-body parameters (set 3) yield the binding energy value which compares favourably with the experimental results.

TABLE 1

	ΣR^2	Triton binding energy
Set 1	4700.8314	7.80
2	4524.29	7784
3	4386.284	7.88

So far as triton binding energy is concerned we see that a separable approximation to the two-body t-matrix can reproduce the binding energy, in addition to its added advantage that it simplifies the problem considerably (Lovelace 1964, Harms *et al* 1969).

The authors are thankful to the authorities of C.M.E.R.I. Durgapur for extending the facility of using IBM 1620.

REFERENCES

Amado R. D. 1963 *Phys Rev* **132**, 548
Amado R. D. & Yam Y. Y. 1965 *Phys. Rev.* **140B**, 1201.
Faddeev L. D. 1961 *Sov Phys JETP* **12**, 1014
Fiedelday H., Erens G., Van wageningen R., Homan D. H. & Kok L. P. 1968 *Nucl Phys.* **A113**, 543.
Harms E. & Lovinger J. S. 1969 *Phys. Rev Lett.* **30B**, 449.
Humberston J. W., Hall R. L. & Osborn T. A. 1968 *Phys. Rev. Lett.* **27B**, 195.
Kharchenko V. F. & Sitenko A. G. 1963 *Nucl. Phys.* **49**, 15.
Kharchenko V. F., Sitenko A. G., Kharchenko V. F. & Petrou N. M. 1966 *Phys. Rev. Lett.* **21**, 54.
Kowalski K. L. *Phys. Rev. Lett.* **15**, 798
Lovelace 1964 *Phys. Rev* **135B**, 1225
Mac Gregor M. H., Arndt R. A. & Wright R. M. 1968 *Phys. Rev.* **173**, 1272.
Malfliet & Tjon 1969 *Nucl Phys.* **A127**, 161.
Mitra, A. N. 1962 *Nucl Phys.* **32**, 529.
Noyes H. P. 1965 *Phys Rev. Lett.* **15**, 538.
Ohmura H. & Ohmura T. 1962 *Phys. Rev.* **128**, 729.
Rarita W. & Present R. D. 1937 *Phys. Rev.* **51**, 788.
Wageningen R. & Kok L. P. 1967 *Nucl. Phys.* **A98**, 303.
Yamaguchi Y. 1954 *Phys. Rev.* **95**, 1628.